Family-Centric School Choice Model with Meaningful Choices and Guaranteed Equal Access to Quality

Peng Shi*, Itai Ashlagi**

MIT Operations Research Center

1 Executive Summary

- By a simultaneous reform in school choice process and algorithm, we can produce a school choice model that
  1. **guarantees equal access to quality** for every student in a quantitatively precise way;
  2. does not use geographic zones but offers each family an **individualized menu of 5-6 options** that while providing district average levels of quality according to multiple metrics, is **optimized to be closest to home**;
  3. **improves predictability** and ease of use by eliminating redundant choices;
  4. **automatically responds to changes** across years in demographics and in school quality.
  5. as much as families' choices allow, **keeps local communities together**.
- If given detailed demand and choice data, we can run simulations to compare with currently proposed options and produce impact report community by community.

2 Outline of Model

Step 1 (**School Evaluation**) Evaluate every school based on a list of quantitative quality metrics. (*i.e.* MCAS composite, DESC level, music program rating, sports rating, facilities rating, etc.)

Step 2 (**Application Phase 1**) Every new prospective family fills out a preliminary BPS application. If the incoming child has currently attending siblings, the family can request assigning the child to those schools. Otherwise, families do not choose in phase 1.

Step 3 (**Assignment Phase 1 - Assign Siblings**) Satisfy as many sibling requests as possible.

Step 4 (**Compute Equitable Choice Options**) Out of the remaining open seats, compute the district average level of quality with respect to each metric. These district averages are the levels of quality we will guarantee (in expectation) for every student. Divide the remaining seats into "probabilistic baskets" for the remaining applicants (*i.e.* one basket might be $\frac{1}{3}$ probability at school A, $\frac{1}{3}$ at B, and $\frac{1}{3}$ at C.). The division is such that the expected values of quality in each basket matches exactly the district average. The schools in a student's basket are the choices offered to this student. These sets of choices are optimized so that they are close to home and have little redundancy (no need to offer same student two schools roughly equivalent in every quality metric). The choice menus are reported to parents via an interactive online tool and/or via mail.

Step 5 (**Application Phase 2 - Families Choose**) Each family reviews the individualized choice menu they receive and submits a ranking to BPS.

Step 6 (**Assignment Phase 2 - Assign Remaining Seats**) An algorithm uses parents' choices to generate final assignment probabilities. (In essence parents "trade probabilities" to improve upon their probabilistic basket of schools.) The final probabilities are implemented in a "community-correlated" lottery, which maximizes chances that children from the same local community attend the same school, wherever that school may be. The assignment results are communicated to parents.

* Ph.D. Candidate, MIT Operations Research Center. pengshi@mit.edu

** Assistant Professor of Operations Management, MIT Sloan School of Management. iashlagi@mit.edu.

3 Description of Each Step

3.1 School Evaluation

An independent committee decides on a list of quantitatively measurable quality metrics. A possible list might be:

Metric:	(MCAS composite,	DESC level,	Music rating,	Sports rating,	general facilities rating)
Scale:	0-100	1-4	1-4	1-4	1-4

Every school is evaluated yearly according to each metric. For school s, let its vector of quality scores be denoted q^s. For example, school A might have scores

$$q^A = (60, 2, 3, 2, 2)$$

and school B might have

$$q^B = (70, 3, 1, 3, 1)$$

The decision of what quality metrics to include and how exactly to measure can be decided by EAC and/or BPS and may be revised without changing the rest of the model.

3.2 Application Phase 1

Every prospective family fills out a simple online (or paper) form applying to BPS for the upcoming school year. They input their home address, whether or not the child has currently attending siblings, and SPED/ELL information. If they have currently attending siblings, they can request to be assigned to those schools, and they would rank those schools in case of multiple siblings in different schools. Otherwise, families do not submit choices in phase 1.

3.3 Assignment Phase 1 - Assign Siblings

Try to satisfy as many sibling requests as possible. To do this in a fair way, order the kids randomly and as we go through the ordering, assign each kid according to his/her sibling preference as long as capacity allows.

This can also be the time to assign SPED/ELL children using a separate process.

3.4 Compute Equitable Choice Options

After sibling assignment, we can equitably divide the remaining seats so that every remaining child is *guaranteed equal access to quality.*

The level of quality guarantee, q^*, is simply the list of averages of the remaining seats' school quality scores. For example, one possible q^* might be

$$q^* = (70, 3, 2, 2, 2)$$

We will guarantee that for every student, unless they choose otherwise, the expected value of the quality of their assigned school according to every metric is at least this.

To do this, we divide remaining seats fractionally, giving each student a "probabilistic basket" of seat assignments. (One such basket may be probability $\frac{1}{3}$ at school A, $\frac{1}{3}$ at B, and $\frac{1}{3}$ at C.) Every basket is such that the expected quality of schools in this basket matches the guaranteed q^*. For example, if $q^* = (70, 3, 2, 2, 2)$, and schools A, B, and C have scores $q^A = (60, 2, 3, 2, 2)$,$q^B = (70, 3, 1, 3, 1)$, and $q^C = (80, 4, 2, 1, 3)$, then the basket with $\frac{1}{3}$ of each of A, B, and C is valid because

$$q^* = \frac{1}{3}q^A + \frac{1}{3}q^B + \frac{1}{3}q^C$$

($70 = \frac{1}{3} \times 60 + \frac{1}{3} \times 70 + \frac{1}{3} \times 80$, $3 = \frac{1}{3} \times 2 + \frac{1}{3} \times 3 + \frac{1}{3} \times 4$, etc.)

Here school A has MCAS score of 60, which is lower than the district average of 70. And school C has higher than average MCAS of 80. But if a student receives each of A, B, C with probability $\frac{1}{3}$, his/her expected value of school MCAS score is exactly the district average of 70. Hence, this basket delivers the guaranteed expected quality level $q_1^* = 70$. For this basket, the same guarantee holds for all the other metrics as well.

There are many probabilistic baskets of schools that deliver the guaranteed levels of quality. Some baskets have higher expected distance from home than others. Some may involve more schools than others. Using a computer algorithm (based on linear programming), we compute the optimal baskets for all remaining students that minimizes total expected distance to home and also minimizes number of schools involved. Moreover, the computed baskets are jointly feasible (can be simultaneously achieved with available supply).

For every student, the set of schools that he/she received a positive probability at is his/her individualized choice menu. (The above optimization results in these options being close to home, meaningful in variety, and fewer in number.)

Details of the above optimization are given in Appendix A.1.

3.5 Application Phase 2 - Families Choose

The computed choice menu for each family is published online (or via mail). If we use k different quality metrics, the algorithm guarantees that on average each parent will have about $k + 1$ choices. (So using 5 quality metrics results in the average family receiving 6 choice options.) Parents research these options and based on their preferences submit a ranking.

3.6 Assignment Phase 2 - Assign Remaining Seats

A modification of the Top Trading Cycle Algorithm used in New Orleans School Choice takes parents' rankings and compute the optimal assignment probabilities. (Essentially, the algorithm allows parents to trade probabilities if it makes everyone better off. For example, in the previous example because school B is better than school C in music and school C is better in sports, two parents who both receive the same basket with $\frac{1}{3}$ at school B and $\frac{1}{3}$ at C might trade with one another so the one who prefers music gets $\frac{2}{3}$ at school B, and the other gets $\frac{2}{3}$ at C.) Details of this procedure is in Appendix A.2

After computing the final assignment probabilities, a centralized lottery algorithm implements these probabilities in a community-correlated way: while the probability of each student going to each school is fixed from above, we maximize the probability that two kids from the same local neighborhood are assigned together. This allows these kids to share rides and/or homework help and maintain community even when kids do not end up going to their neighborhood school. (The same technique can be used to improve community cohesion in any model, so this procedure can be adopted independently.) Details of how this procedure works is in Appendix A.3

The final assignment is communicated to parents.

4 Pros and Cons of Proposed Model

Pros:

- Guarantees equal access to quality for every student in a measurable way.
- Fewer but more meaningful choice options.
- Greater predictability in assignment outcome (due to fewer choice options).
- Built-in response to changes in demographics and supply across years. No need to update zones when people move or qualities changes.
- Minimizes total distance to school while delivering equal access to quality. If BPS one day achieves every neighborhood school delivering equal levels of quality, then the model becomes simply the ideal neighborhood model. Hence this has an add-on feature of increasing BPS' financial incentives to distribute quality evenly (to save busing costs).
- Maintains strategyproofness (families have no incentives to lie about their preference rankings.)

Cons:

- Requires 2 phases of application for families.
- Departure from publicly accepted zone approach.
- Greater complexity in the back-end computer algorithm. (But to everyone but the programmer this complexity is hidden so should not make a difference.)
- A plan with individualized (or geocode based) assignment options may be harder to present than simple zones. Requires online interactive application and/or mail to parents.
- For the same local neighborhood, choice options might change from year to year. (But if supply, demand, and qualities do not change too much, the change should be minimal.)

5 Data Request

If given anonymized demand data along with families' choices, we can run simulations and quantitatively evaluate the impact of this model, and compare with other proposed models, giving detailed neighborhood by neighborhood reports. We can also perform more sophisticated analysis if requested.

A Technical Details

The following details are relevant only for the programmer. Parents and BPS staff are shielded from this complexity.

In the following, let n be the number of students and m the number of schools.

A.1 Algorithm to Compute Optimal Choice Options

Given supply data (after siblings have been assigned), demand data (applicants and their locations) and school quality data (list of k quality metrics for each school), we compute using linear programming a basket of guaranteed assignment probabilities for each applicant, such that the basket for each student achieves the district average quality for each metric and the total expected distance from home is minimized.

For every incoming child i and school s, let d_{is} be the distance between the child's home and the school. Let q^s be the k-dimensional vector of quality scores for school s. Let c_s be the remaining capacity for incoming students at school s. Define variable x_{is} to be the probability of student i receiving a spot at school s. We solve the following linear program (can be done in a few seconds on a computer)

$$\begin{array}{lll} \text{Minimize} & \sum_{is} d_{is} x_{is} & \text{expected distance to home} \\ \text{Subject to} & \sum_s q^s x_{is} \geq q^* \ \forall i & \text{quality guarantee} \\ & \sum_s x_{is} = 1 \ \ \forall i & \text{every student is assigned} \\ & \sum_i x_{is} \leq c_s \ \forall s & \text{supply satisfies demand} \\ & x_{is} \geq 0 & \end{array}$$

There exists an optimal solution x^* with at most $n(k+1)+m$ non-zero x^*_{is}'s, so it is possible to offer each student i an optimal probabilistic basket of about $k+1$ schools on average, such that the expected quality is at least the guaranteed level q^*. The set $S(i) = \{s : x_{is} > 0\}$ is the individualized choice menu for student i, and $\{x^*_{is}\}$ is the guaranteed levels of access.

A.2 Algorithm to Compute Optimal Choice Options

Given each applicant i's ranking $\succ_i$ over choice menu $S(i)$, and given the guaranteed access probabilities x^*_{is} (from Section A.1), we compute the final assignment probabilities $\{p_{is}\}$ by averaging the results of several runs of a variant of the Top-Trading-Cycle Algorithm currently implemented in New Orleans School Choice.

One possible description of the Top Trading Cycle Mechanism used in New Orleans in the case without priorities is as follows: tentatively assign children to school seats uniformly randomly. Within each school, uniformly randomly rank the students tentatively assigned. Let the top ranked student in each school point to his/her favorite unfilled school. There is at least one cycle. Allow the kids in this cycle to trade their spots for their favorite unfilled school, and finalize these assignments. Remove these kids from consideration and update the corresponding schools' capacities and list of tentative students. The procedure continues until all seats are assigned.

In our setting, we cannot simply apply the above because we need to guarantee every student i an assignment probability x^*_{is} to school s unless he/she chooses to trade part of this probability for increased probability at a school he/she prefers more. However, the following variant accomplishes this: for each student i randomly assign him/her a seat at school s with probability x^*_{is}, and do this independently for all students. Then, order the students assigned in each school according to a uniformly random ordering, and perform the same Top Trading Cycle algorithm as above. We compute the final assignment probabilities p_{is} simply by repeating the above process many times and averaging the assignment frequencies.

Technical note: because of the independent randomization in assigning tentative choices above, in each particular instance some schools' capacities might be violated. However, this problem disappears by the Law of Large Numbers as we are averaging across many simulation runs. So the final p_{is}'s do not violate any school's capacity.[1]

A.3 Algorithm to Correlate Lottery to Keep Communities Together

Given the assignment probabilities p_{is} (from Section A.2), we compute a lottery implementation that maximizes the chance that kids from the same local community are assigned to the same school. (To define "local community," we can use students' geocode, block, or some other partition.)

Define X as the space of random assignments that assign every student and do not violate any school capacities.[2] For any random assignment $x \in X$, define $E[f(x)]$ as the expected number of pairs of kids from

[1] More precisely, there might be small errors but by averaging enough runs we can make errors as small as we like.

[2] More precisely, X is the set of distributions over vertices of the assignment polytope: $\mathcal{P} = \left\{a \in \mathbb{R}^{n \times m} : \sum_{s \in S} a_{is} = 1, \sum_{i \in I} a_{is} \leq c_s, 0 \leq a_{is} \leq 1\right\}$

the same community assigned to the same school. We solve the following mathematical program

$$\begin{aligned} \text{Max} \quad & E[f(x)] \\ \text{s.t.} \quad & E[x] = p \\ & x \in X. \end{aligned}$$

While it is computationally hard to solve this exactly, there are many possible algorithms to find a good solution practically. One such algorithm, which finds a solution to full-sized problems in minutes, is documented in [1]. For more details on this procedure, please refer to [1].

References

1. Peng Shi and Itai Ashlagi. Improving community cohesion in school choice via correlated-lottery implementation. http://www.mit.edu/~pengshi/papers/community-cohesion.pdf, 2012.